

Baryon Axial Charges and Momentum Fractions with $N_f = 2 + 1$ Dynamical Fermions

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We report on recent results of the QCDSF/UKQCD Collaboration on investigations of baryon structure using configurations generated with $N_f = 2 + 1$ dynamical flavours of $O(a)$ improved Wilson fermions. With the strange quark mass as an additional dynamical degree of freedom in our simulations we avoid the need for a partially quenched approximation when investigating the properties of particles containing a strange quark, e.g. the hyperons. In particular, we will focus on the nucleon and hyperon axial coupling constants and quark momentum fractions.

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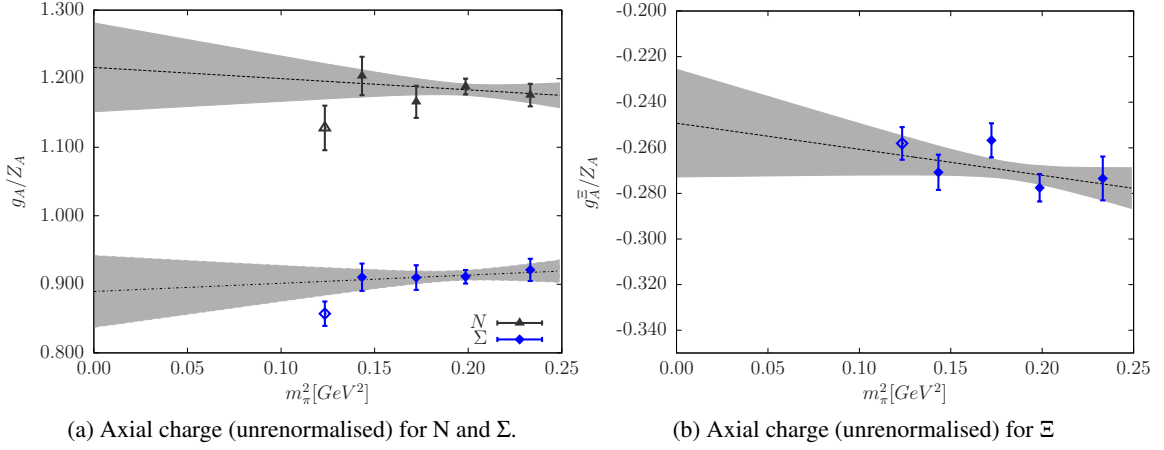


Figure 1: Unrenormalised baryon axial charge. The data points depicted with open symbols were omitted for the two-parameter linear fits.

1. Introduction

The nucleon axial charge governs neutron β -decay and also provides a quantitative measure of spontaneous chiral symmetry breaking. It is related to the first moment of the helicity dependent quark distribution functions, $g_A = \Delta u - \Delta d$, and has been studied theoretically as well as experimentally for many years. Experiment determine its value, $g_A = 1.2695(29)$, to high accuracy. Hence it is an important quantity to study on the lattice, and since it is relatively clean to calculate (zero momentum, isovector), it serves as a useful yardstick for lattice simulations of nucleon structure.

While there has been much work on the (experimentally well-known) nucleon axial charge g_A , there have only been a handful of lattice investigations of the axial charge of the other octet baryons g_A^B [1–3], which are relatively poorly known experimentally. These constants are important since at leading order of SU(3) heavy baryon chiral perturbation theory, these coupling constants are linear combinations of the universal coupling constants D and F , which enter the chiral expansion of every baryonic quantity.

Much of our knowledge about QCD and the structure of the nucleon has been derived from deep inelastic scattering (DIS) experiments where cross sections are determined by its structure functions. The DIS structure functions are related to parton distribution functions (PDF) by asymptotic free constituents in the parton model. Through the operator product expansion, moments of DIS structure functions or moments of PDFs are related to matrix elements of towers of twist-2 operators (and Wilson coefficients which are calculable in perturbative QCD).

While the quark momentum fractions of the nucleon and pion have received much attention for many years (see, e.g., [4] for a recent review), there have to date been no investigations of the flavour SU(3) symmetry breaking effects of the quark momentum fractions of the hyperons. The obvious question that arises in this context is: “How is the momentum of the hyperon distributed amongst its light and strange quark constituents?”

κ_l	κ_s	$m_\pi[GeV]$	$m_K[GeV]$	$N \times N_T$	$m_\pi L$	N_{meas}
0.120830	0.121040	0.481	0.420	24x48	4.63	2500
0.120900	0.120900	0.443	0.443	24x48	4.28	4000
0.120950	0.120800	0.414	0.459	24x48	3.99	2500
0.121000	0.120700	0.377	0.473	24x48	3.63	2500
0.121040	0.120620	0.350	0.485	24x48	3.37	2500

Table 1: Simulation parameters for $N_f = 2 + 1$ dynamical fermions with two mass-degenerate light quarks and one strange quark. The simulation parameter β was chosen to $\beta = 5.50$ which corresponds to a lattice spacing of $a = 0.078(3)\text{fm}$.

We present preliminary results from the QCDSF/UKQCD Collaboration for the octet hyperon axial charge, g_A^B , and quark momentum fraction, $\langle x \rangle_q^B$, for $B = \{N, \Sigma^+, \Xi^0\}$ determined with lattice QCD simulations with $N_f = 2 + 1$ flavours of dynamical $O(a)$ improved Wilson fermions.

2. Simulation Details

Our gauge field configurations have been generated with $N_f = 2 + 1$ flavours of dynamical fermions, using the Symanzik improved gluon action and nonperturbatively $O(a)$ improved Wilson fermions [5]. We choose our quark masses by first finding the flavour SU(3)-symmetric point where flavour singlet quantities take on their physical values and vary the individual quark masses while keeping the singlet quark mass $\bar{m}_q = (m_u + m_d + m_s)/3 = (2m_l + m_s)/3$ constant [6]. Simulations are performed on lattice volumes of $24^3 \times 48$ with lattice spacing, $a = 0.078(3)\text{fm}$. A summary of the parameter space spanned by our dynamical configurations can be found in Table 1.

3. Baryon Axial Charge g_A^B

The axial charge is defined as the axial vector form factor at zero four-momentum transfer, $g_A = G_A(0)$, which is obtained from the matrix element for the baryon, B

$$\langle B(p', s') | A_\mu^{u-d} | B(p, s) \rangle = \bar{u}_B(p', s') \left[\gamma_\mu \gamma_5 G_A(q^2) + \gamma_5 \frac{q_\mu}{2m_N} G_P(q^2) \right] u_B(p, s), \quad (3.1)$$

where $q = p' - p$ denotes the 4-momentum transfer and $u_B(p, s)$ is the spinor for the baryon, B , with momentum p and spin vector s and G_P is the induced pseudoscalar form factor. The isovector axial current is defined as $A_\mu^{u-d} = \bar{u} \gamma_\mu \gamma_5 u - \bar{d} \gamma_\mu \gamma_5 d$ where u and d denote the up and down quark fields, respectively. We work in the limit of exact isospin invariance, i.e. u and d quarks are assumed to be degenerate in mass. The states are normalised according to $\langle p', s' | p, s \rangle = (2\pi)^3 2p^0 \delta(\mathbf{p} - \mathbf{p}') \delta_{ss'}$, we take $s^2 = -m_B^2$ and m_B is the baryon mass. Thus the axial charge is given by the forward matrix element $\langle B(p, s) | A_\mu^{u-d} | B(p, s) \rangle = 2g_A^B s_\mu$. In parton model language, the forward matrix elements of the axial current are related to the fraction of the spin of the baryon carried by the quarks. Denoting by $\langle 1 \rangle_{\Delta q}^B$ the contribution of the quark, q , to the spin of the baryon, B , one finds

$$\langle B(p, s) | \bar{q} \gamma_\mu \gamma_5 q | B(p, s) \rangle = 2 \langle 1 \rangle_{\Delta q}^B s_\mu. \quad (3.2)$$

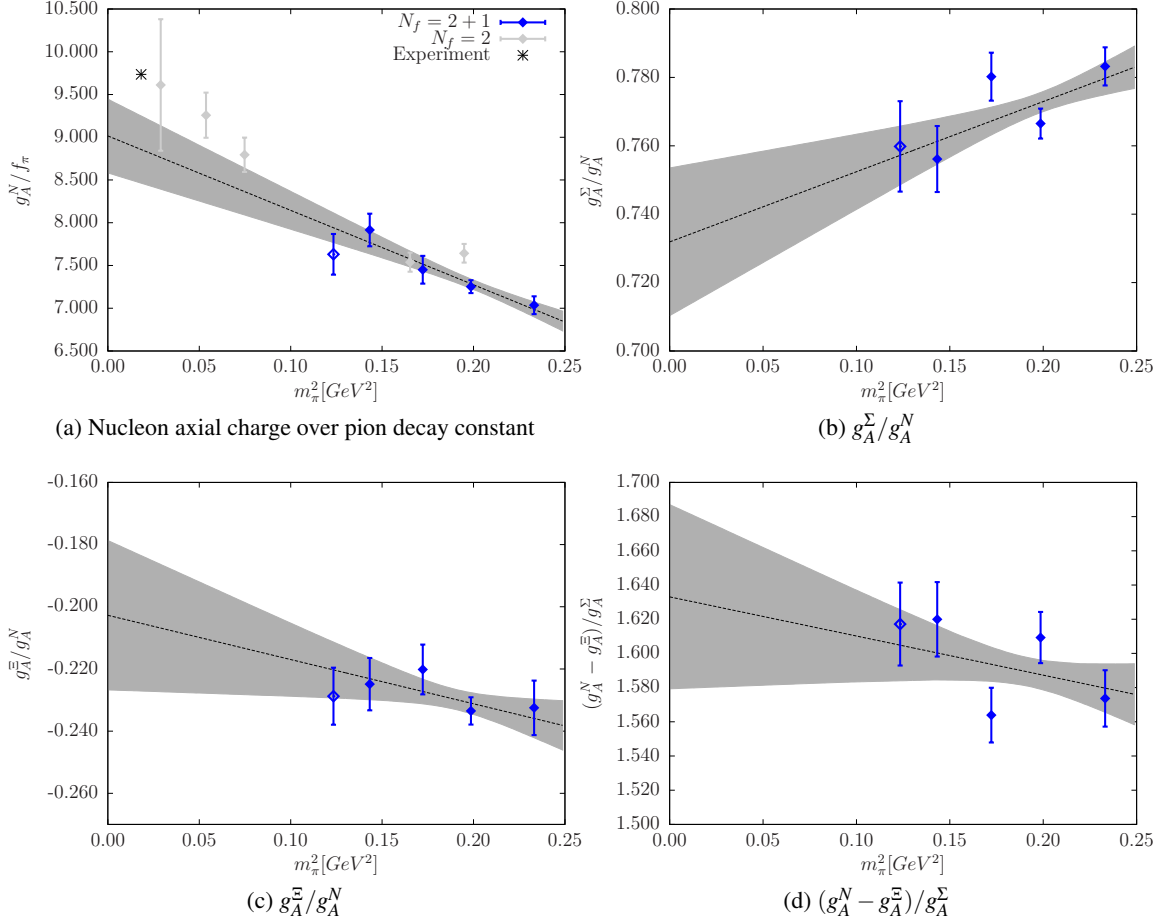


Figure 2: Ratios of unrenormalised baryon axial charge, the renormalisation constant cancels.

Thus for the nucleon we write $g_A^N = \langle 1 \rangle_{\Delta u}^N - \langle 1 \rangle_{\Delta d}^N$.

Figure 1a (1b) shows the unrenormalised axial charge for the nucleon and the Σ (Ξ). It is well known that the nucleon axial charge is sensitive to finite size effects (FSE) [7]. We suppose that the drop of g_A^B for $B = \{N, \Sigma\}$ at the lightest pion masses is due to FSE.

With the current data we can not do better than a first approximation with a linear two-parameter fit to find the unrenormalised axial charge at the physical point.

The next step would be to renormalise our results, however as yet Z_A is unknown for these ensembles so we instead consider ratios where the renormalisation constant cancels. The first ratio we take is g_A^N / f_{π^\pm} , the nucleon axial charge over the pion decay constant, shown in Fig. 2a. Since the renormalisation constants cancel in the ratio, we are able to compare our results to the experimental value [8] and to our $N_f = 2$ results [9]. Except for the lightest pion mass, which is possibly due to FSE, the measurements show a trend towards the experimental value and agree very well with the $N_f = 2$ results.

R	a_0	a_1	χ^2/dof	quality	value
g_A^Σ/g_A^N	0.732(22)	0.21(11)	6.043090	0.002374	0.736(22)
g_A^Ξ/g_A^N	-0.203(24)	-0.14(13)	1.299522	0.272662	-0.205(24)
$(g_A^N - g_A^\Xi)/g_A^\Sigma$	1.633(54)	-0.23(28)	6.498347	0.001506	1.629(54)
$(g_A^N + g_A^\Xi)/g_A^\Sigma$	1.082(45)	-0.45(23)	0.311016	0.577057	1.074(45)

Table 2: Ratios of the baryon axial charge in the chiral limit. Extrapolations to the physical point were obtained via a two-parameter linear fit model $R = a_0 + a_1 m_\pi^2$.

4. Ratios of the Axial Charge

In the case of exact flavour SU(3) symmetry, the axial charge of the N , Σ , and Ξ ground states are connected by the following linear combinations of the SU(3) constants, F and D [10, 11]

$$g_A^N = F + D \quad g_A^\Sigma = 2F \quad g_A^\Xi = D - F.$$

We consider ratios of the baryon axial charge in which the renormalisation constant cancels:

$$\frac{g_A^\Sigma}{g_A^N} = \frac{2F}{F+D} \quad \frac{g_A^\Xi}{g_A^N} = \frac{F-D}{F+D} \quad \frac{g_A^N - g_A^\Xi}{g_A^\Sigma} = \frac{D}{F} \quad \frac{g_A^N + g_A^\Xi}{g_A^\Sigma} = 1$$

Figs. 2b - 3a show these ratios as a function of m_π^2 . We linearly extrapolate to the physical quark mass to obtain preliminary predictions which we collectively show in Table 2. These preliminary results are in good agreement with earlier lattice [1, 3] and quark model [12] determinations.

From a fit to the experimental data taking model independent leading SU(3) breaking contributions to the axial current matrix elements into account Savage and Walden [13] found the following values: $F = 0.47(7)$ and $D = 0.79(10)$. Combining the central values the following ratios are obtained: $g_A^\Sigma/g_A^N = 0.75$, $g_A^\Xi/g_A^N = -0.25$, and $(g_A^N - g_A^\Xi)/g_A^\Sigma = 1.68$. Thus, our results are in good accordance with their results.

5. Momentum Fractions

The first moment of a baryon's, B , unpolarised quark distribution function, $q(x)$ gives the total fraction of the baryon's momentum carried by the quark, q , $\langle x \rangle_q^B$. This moment is related to the matrix element of a twist-2 operator (and a Wilson coefficient)

$$\langle B(p) | \bar{q} \gamma^{\{\mu} i \overleftrightarrow{D}^{\nu\}} q | B(p) \rangle = 2 \langle x \rangle_q^B p^{\{\mu} p^{\nu\}}, \quad (5.1)$$

where $\overleftrightarrow{D} = (\overrightarrow{D} - \overleftarrow{D})/2$ is the forward/backward covariant derivative.

We determine the individual connected quark contributions, $\langle x \rangle_q^B$, and take the difference of the doubly and singly represented quark contributions ($(D - S)$, which is $(u - d)$ in the nucleon) so that the disconnected contributions cancel. As in the previous section for the axial charge, the renormalisation constant for the momentum fractions $Z_{v_{2b}}$ has not yet been determined, and so

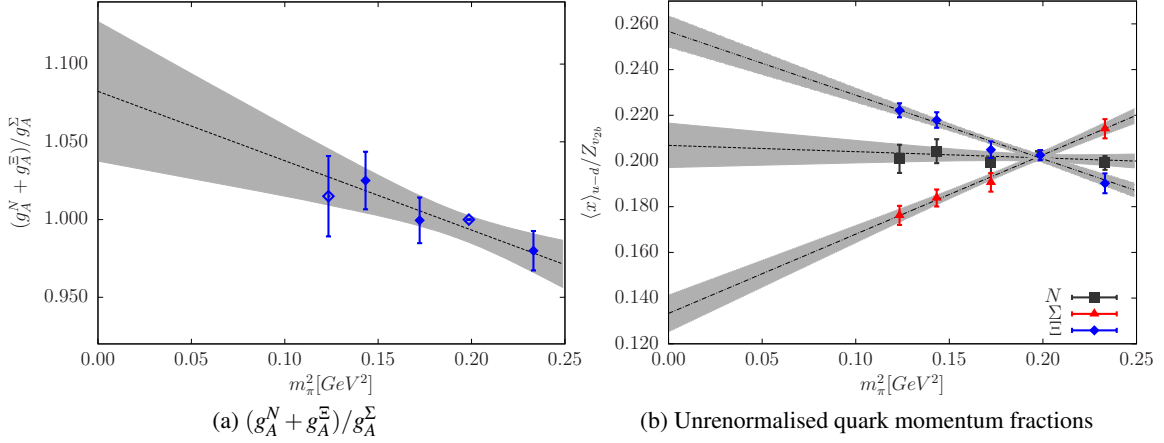


Figure 3: Left side: Ratio of baryon axial charge. Right side: Quark momentum fractions for the N , Σ , and Ξ .

we are not yet able to make any quantitative predictions for the quark momentum fractions of the hyperons.

Figure 3b shows $(D - S)$ quark momentum fractions for the nucleon ($u - d$), $\Sigma(u - s)$, and $\Xi(s - u)$. Here we see clear evidence for flavour SU(3)-symmetry breaking effects as the fractions carried by the light and strange quark fan out from the symmetric point as we decrease the pion mass. This result indicates that the larger contribution to the baryon momentum is carried by the (heavier) strange quark, and that this contribution increases (and in turn, the light quark contribution decreases) as the strange (light) quark mass is increased (decreased) towards its physical value.

6. Conclusions

We have presented preliminary results from the QCDSF/UKQCD Collaboration for the axial charge and quark momentum fraction of the octet baryons N , Σ , and Ξ from lattice QCD simulations with $N_f = 2 + 1$ flavours of dynamical fermions.

Our results for the hyperon axial charge agree well with earlier lattice results and show a hint of flavour SU(3)-symmetry breaking effects. The quark momentum fractions of the octet hyperons, on the other hand, show strong flavour SU(3)-symmetry breaking effects, with the heavier strange quark contributing a larger fraction to the total baryon momentum than the light quarks.

An obvious feature that is currently lacking from these results is a determination of the renormalisation constants for the local operators considered here. These calculations are now underway and will allow us to make more quantitative predictions in the near future.

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